Influence of absorption on the zero-$\bar{n}$ gap in one-dimensional photonic crystals with left-handed materials

M. de Dios-Leyva and J. A. Leyva-Galano
Department of Theoretical Physics, University of Havana, San Lázaro y L, Vedado 10400, Havana, Cuba
(Received 20 May 2008; published 8 September 2008)

We have investigated the effects of absorption on the properties of the zero-$\bar{n}$ gap in one-dimensional photonic crystals consisting of alternating layers of air and a metal-based left-handed material (LHM). Calculations are performed by using two different models for the dielectric permittivity and magnetic permeability of the LHM component. It is shown that the dispersion curves and the corresponding density of states around the zero-$\bar{n}$ gap, which is insensitive to the geometrical scaling of the structure in the absence of absorption, are drastically modified by these effects. Specifically, it is demonstrated that absorption creates photon states inside the gap and the resulting band structure exhibits a band gap of zero width. The equation determining the frequency at which the corresponding bands touch each other is reported. Another consequence of these effects is that, for weak and moderate absorption, both the dispersion relation and the density of states exhibit a pseudogap structure which tends to disappear in the limit of strong absorption.

DOI: 10.1103/PhysRevB.78.115106 PACS number(s): 42.70.Qs, 78.20.Ci, 41.20.Jb

I. INTRODUCTION

The propagation of electromagnetic waves in regular arrays of materials with different refractive indices, commonly called photonic crystals,[1] has received much attention in the last two decades. This interest is motivated by the interesting basic electromagnetic properties of these structures and by their potential applications in a wide range of optical devices. These applications are intimately related to the existence of photonic band gaps in the dispersion relation of such structures which allows the active control and manipulation of the properties of electromagnetic radiation.[2] It is then very important to study in detail those situations and phenomena that can affect the properties of such gaps. In that respect, it is well known that conventional photonic band gaps originating from the interference of Bragg scattering are highly sensitive to the lattice constant, incident angle, and disorder. This may substantially limit the use of Bragg gaps for the control and manipulation of electromagnetic radiation.

One way to overcome some of these limitations is to realize photonic crystals containing left-handed metamaterials (see Ref. 3 and references therein). This suggestion is based on the following observations. The main physical property of these metamaterials is that both the dielectric permittivity $\varepsilon(\omega)$ and magnetic permeability $\mu(\omega)$ are negative in some frequency ranges; as a result, they also exhibit a negative index of refraction. Due to this, left-handed metamaterials exhibit unusual physical properties, some of which, such as inverse Snell's law, reversed Doppler and Cherenkov effects, and a Poynting vector directed opposite to the wave vector, were discussed by Veselago in his 1968 work.[4] It has been shown[5–11] that one-dimensional (1D) photonic crystals composed of alternating layers of right-handed material (RHM) and left-handed material (LHM) exhibit new types of electromagnetic properties that do not exist in ordinary structures constituted only of RHM. In particular, it was shown[5] the existence of a new type of band gap that corresponds to the frequency at which the spatial average of the refractive index, taken over a period of the 1D photonic structure, vanishes. In contrast with the Bragg gap, such a zero-$\bar{n}$ gap is insensitive to the geometrical scaling of the structure and to structural disorder that is symmetric in the random parameters a and b, where a and b are the widths of the RHM and LHM layers, respectively.

It should be pointed out that these properties of the zero-$\bar{n}$ gap were established in the absence of losses. However, as shown for metamaterials based on wires and split ring resonators,[12–14] which are characterized by large losses that are mainly due to the optical losses in their metallic components, left-handed metamaterials are by their nature necessarily dispersive and dissipative (see Ref. 3). This means that, in the study of the properties of the zero-$\bar{n}$ gap, it is necessary to take into account effects of absorption. These observations have motivated us to study theoretically such effects, which, up to now, have not received special attention.

Finally, it should be pointed out that the influence of absorption on the properties of conventional photonic crystals has been investigated by various authors. Of course, in order to study these effects, it is necessary to consider photonic crystals fabricated with absorbing materials, such as metals. In correspondence with this, absorbing materials, such as metallic roads[15–17] and spheres,[18] as well as GaAs cylinders[19] embedded in a host dielectric medium have been used in such studies.

The paper is organized as follows. In Sec. II, we detail the theoretical approach. Section III is concerned with the results and discussion. Finally, our conclusions are given in Sec. IV.

II. THEORETICAL FRAMEWORK

The photonic structure we consider here consists of alternating and homogeneous layers of materials A and B, periodically distributed along the z axis, and with dielectric permittivity and magnetic permeability given by $\varepsilon_1(\omega)$, $\mu_1(\omega)$ and $\varepsilon_2(\omega)$, $\mu_2(\omega)$, respectively. We choose the origin of coordinates to be at the center of a given A layer and define $d=a+b$ as the period of the corresponding photonic crystal, where a and b are the widths of layers A and B, respectively.

In what follows, we focus on the normal propagation of a monochromatic electromagnetic wave through the considered photonic crystal. Under these conditions, the dispersion relation of such a structure can be obtained by solving the equation for the amplitude $E(z)$ of the electric field,[5,20]

$$-\frac{1}{\varepsilon(z)}\frac{d}{dz}\left[\frac{1}{\mu(z)}\frac{dE(z)}{dz}\right]=\frac{\omega^2}{c^2}E(z), \tag{1}$$

where $\varepsilon(z)=\varepsilon(z+d)$ and $\mu(z)=\mu(z+d)$ are the periodic dielectric permittivity and periodic magnetic permeability, respectively. For simplicity, in Eq. (1) we have omitted the explicit dependence of these optical parameters on the frequency. Thus, the continuity of the two-component function,

$$\Psi(z)=\begin{bmatrix} E(z) \\ \dfrac{1}{\mu(z)}\dfrac{dE(z)}{dz} \end{bmatrix}, \tag{2}$$

and the Bloch condition,

$$\Psi(z+d)=e^{ikd}\Psi(z), \tag{3}$$

lead immediately to the following expression for the dispersion relation:[20]

$$\cos kd=f(\omega), \tag{4}$$

where

$$f(\omega)=\cos aQ_1\cos bQ_2-\frac{1}{2}\left\{\frac{\mu_1}{\mu_2}\frac{Q_2}{Q_1}+\frac{\mu_2}{\mu_1}\frac{Q_1}{Q_2}\right\}\times\sin aQ_1\sin bQ_2, \tag{5}$$

$Q_1=\frac{\omega}{c}\sqrt{\varepsilon_1(\omega)\mu_1(\omega)}$, $Q_2=\frac{\omega}{c}\sqrt{\varepsilon_2(\omega)\mu_2(\omega)}$, and $n_i=\sqrt{\varepsilon_i(\omega)}\sqrt{\mu_i(\omega)}$, with $i=1,2$, are the corresponding refractive indices; k in Eqs. (3) and (4) is the Bloch wave vector along the axis of the photonic crystal that relates the electric field in two consecutive unit cells by the factor $\exp(ikd)$.

Now, the properties of the relation between the wave vector k and frequency ω determined by Eq. (4), as well as of the corresponding Bloch modes, depend on the geometrical and physical parameters characterizing the considered photonic crystal, i.e., on the physical conditions under which these modes propagate. In fact, in the absence of losses the solutions of Eq. (4) determine the band structure of the considered system with allowed zones separated by band gaps. If ω is inside an allowed band, k is real and the corresponding Bloch modes propagate along the axis of the photonic crystal. Inside a band gap, k is complex and an electromagnetic wave cannot propagate in the system. As is well known, the effects of losses can be considered by including the imaginary part of the dielectric permittivity and magnetic permeability in the calculations. This means that $f(\omega)$ in Eq. (4) is complex and therefore the wave vector k and the frequency ω are in general complex. Writing

$$\text{(a)}\ k=k_1+ik_2,\quad \text{(b)}\ \omega=\omega_1-i\omega_2, \tag{6}$$

the imaginary parts k_2 of k and ω_2 of ω determine, respectively, the attenuation length l and the lifetime τ of the modes according to the following definitions:

$$\text{(a)}\ \frac{1}{l}=2k_2,\quad \text{(b)}\ \frac{1}{\tau}=2\omega_2, \tag{7}$$

where $\omega_2\geq 0$.

In accordance with the above discussion, in the study of the effects of losses on the properties of Bloch modes, it is necessary to establish clearly the exact correspondence between the physical conditions under which these modes propagate and the mapping determined by Eq. (4) between the points in the k and ω complex planes. In that respect, two different physical approaches or mapping have been considered.[15,21] In one of them, the wave vector k is assumed to be real and the frequency ω complex. Physically, this approach corresponds to the situation where an arbitrary Bloch mode is excited, acquiring a finite lifetime, and then it decays as time progresses. In the other one, the frequency ω is real and the wave vector k is in general complex. In this case, the Bloch mode at a given ω is attenuated as it propagates through the photonic crystal, and the attenuation length l in Eq. (7) accounts for the effects of absorption and/or photonic band gaps. Note that the latter effect is also present in the no absorbing case.

III. RESULTS AND DISCUSSION

In order to study the effects of absorption on the properties of the zero-$\bar{n}$ gap, we focus on RHM-LHM photonic crystals for which the constituting materials are air (layer A, with $\varepsilon_1=1$ and $\mu_1=1$) and a metal-based LHM (layer B). It is well known that, in the study of these metamaterials, the Lorentz model and its derivatives as well as the Drude model have been used (see Ref. 22 and references therein). These studies indicate that both models may be used for modeling the properties of these structured materials. Based on these observations, we first consider the metal-based LHM of the considered photonic crystals modeled via the lossy Drude polarization and magnetization models. This means that the permittivity and permeability of layer B follow the plasma-like dispersion,

$$\varepsilon_2(\omega)=\varepsilon_0-\frac{\omega_p^2}{\omega(\omega+i\Gamma_p)}, \tag{8}$$

$$\mu_2(\omega)=\mu_0-\frac{\omega_m^2}{\omega(\omega+i\Gamma_m)}, \tag{9}$$

where ε_0, μ_0, ω_p, and ω_m are material parameters, $\omega=2\pi\nu$ is the angular frequency measured in gigahertz, and Γ_p and Γ_m are absorption constants. It should be pointed out that, in the absence of absorption, some authors[8,20] have studied the properties of this type of 1D photonic crystals, including their band structure and the zero-$\bar{n}$ gap. In our theoretical calculations, we take $\varepsilon_0=1.21$, $\mu_0=1$, $\omega_p=\omega_m=10$ GHz and suppose, for simplicity, that $\Gamma_p=\omega_p\gamma$ and $\Gamma_m=\omega_m\gamma$, where γ is a dimensionless absorption parameter.

Let us now consider the solutions of Eq. (4), which may be rewritten as

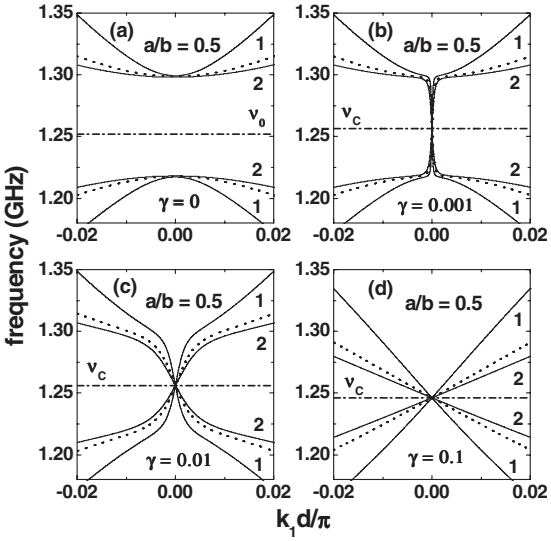

FIG. 1. Photonic band structure of a 1D photonic crystal composed of alternating layers of air and a LHM with $\varepsilon_2(\omega)$ and $\mu_2(\omega)$ given by Eqs. (8) and (9), respectively, for $a/b=0.5$, with $a=6$ mm (solid lines 1), $a=12$ mm (dotted lines), and $a=16$ mm (solid lines 2), and for different values of the absorption parameter γ. The frequency $\nu=\omega/2\pi$ is presented as a function of $\tau=k_1d/\pi$ for the two bands associated with the corresponding zero-$\bar{n}$ gap. ν_C satisfies Eq. (11) and ν_0 is the frequency at which the corresponding zero-$\bar{n}$ gap opens.

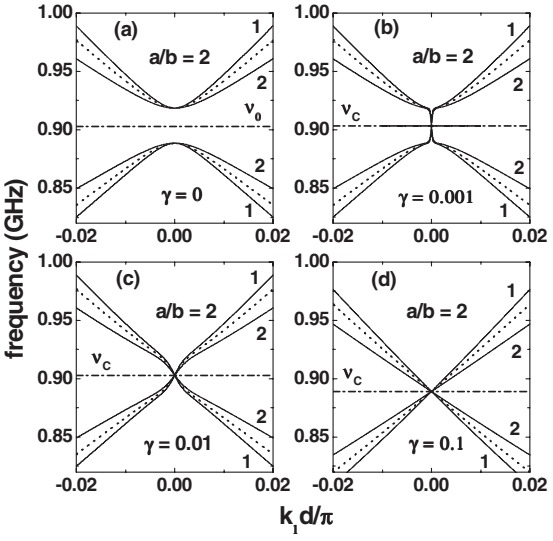

FIG. 2. The same as Fig. 1, except that $a/b=2$, with $a=12$ mm (solid lines 1), $a=14$ mm (dotted lines), and $a=18$ mm (solid lines 2).

$$\cos(k_1 + ik_2)d = f_1(\omega) + if_2(\omega), \tag{10}$$

for ω real and $k=k_1+ik_2$ complex, where $f_1(\omega)$ and $f_2(\omega)$ are, respectively, the real and imaginary parts of the function $f(\omega)$ in Eq. (4). According to Eq. (10), the study of the Bloch modes in this approach reduces to the study of the solutions of a set of two transcendental equations for three variables k_1, k_2, and ω. Of course, only one of them is independent. Taking k_1 as the independent variable, the relations $\omega(k_1)$ and $k_2(k_1)$ determine, respectively, the dispersion relation of the photonic crystal and the dependence on k_1 of the corresponding attenuation length. The real part k_1 of the Bloch wave vector k is restricted to the first Brillouin zone $-\pi/d<k_1<\pi/d$.

In order to illustrate the effects of absorption on the zero-$\bar{n}$ gap, we have calculated the two photonic bands associated with such a gap for two different values of the ratio a/b and various values of the width a of layer A (air) and of the absorption parameter γ. Figures 1 and 2 display a magnification of these bands as a function of the reduced Bloch wave vector $\tau=k_1d/\pi$. As $\phi=b/d=1/(1+a/b)$ determines the filling fraction of the LHM layer as a function of the ratio a/b, it takes the values $\phi=2/3$ and 1/3 in Figs. 1 and 2, respectively.

We first note that, ignoring quantitative difference, the results depicted in Fig. 1, for $a/b=0.5$, are very similar to those displayed in Fig. 2, for $a/b=2$. The quantitative difference between these results is mainly determined by effects associated with the filling fraction. Note, in particular, that these effects directly affect both the width and center of the band gap, as well as the degree of curvature of the dispersion relation [see Figs. 1(a) and 2(a)]. Now, in panels 1(a) and 2(a) we present the mentioned band structure in the absence of absorption ($\gamma=0$). In each one of these figures we have also plotted a line indicating the frequency $\nu_0=\omega_0/2\pi$ at which the corresponding zero-$\bar{n}$ gap opens.[5] As expected, the zero-$\bar{n}$ gap is insensitive to the geometrical scaling of the structure.

Further, when absorption is taken into account, the $\bar{n}=0$ condition cannot be satisfied and instead of a band structure possessing a zero-$\bar{n}$ gap another type of band structure should form. As shown in Figs. 1 and 2, the structure exhibits a band gap of zero width for any $\gamma\neq 0$. It follows from Eq. (10) that touching the corresponding bands occurs at the center of the Brillouin zone ($\tau=0$) and at the frequency $\nu_C=\omega_C/2\pi$ for which

$$f_2(\omega_C) = 0. \tag{11}$$

As shown in Figs. 1 and 2, the frequency ν_C is practically independent of the width a, for given values of the ratio a/b and of the absorption parameter γ, and it is a slowly varying function of γ, for a fixed value of the ratio a/b. Moreover, it is clearly seen that the properties of the dispersion curves are drastically modified by absorption, especially in the case of moderate and strong absorptions. Note, in particular, that touching the bands at $\tau=0$ occurs with finite derivative $d\omega/d\tau$, i.e., the group velocity $v_g(\tau)$ at $\tau=0$ is different from zero for any finite value of the absorption parameter γ. Also, it follows from the numerical results depicted in Figs. 1 and 2 that, for a given value of the Bloch wave vector τ near the center of the Brillouin zone, the group velocity $v_g(\tau)$, which is determined by the slope of the dispersion curve, decreases with the absorption parameter γ. These results indicate that the density of states (DOS) $\rho(\omega)$, which is proportional to the reciprocal of the group velocity, is a finite and increasing function of γ for ω near ω_C [see Eq. (11)]. As a result, absorption creates photon states inside the band gap, giving

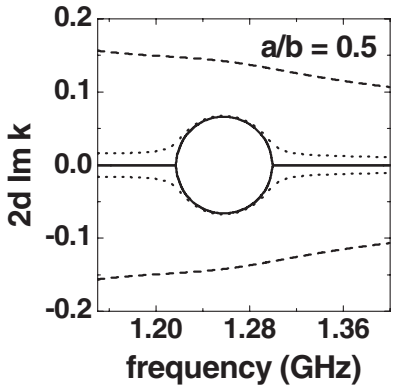

FIG. 3. Imaginary part $2dk_2$ of the wave vector $2dk$ as a function of the frequency $\nu=\omega/2\pi$ for $a/b=0.5$ ($a=6$ mm) and for $\gamma=0$ (solid lines), $\gamma=0.01$ (dotted lines), and $\gamma=0.1$ (dashed lines).

rise to a redistribution of such states in the system due to the conservation of the number of total photon states.

Furthermore, it is obvious that, for a better understanding of the above results, it is necessary to know the asymptotic solution of Eq. (10) for small values of the reduced Bloch wave vector τ and for ω near ω_C. In order to obtain such a solution, we first focus on the behavior of the imaginary part $k_2(\omega)$ of k around the frequency $\nu_C=\omega_C/2\pi$. In Fig. 3, the dimensionless quantity $2dk_2(\omega)$, which is the reciprocal of the attenuation length (in units of the period d), is depicted as a function of the frequency $\nu=\omega/2\pi$ for a given value of the ratio a/b and various values of the absorption parameter γ. It follows from Fig. 3 that the properties of the attenuation length are dramatically modified by absorption, especially for moderate and strong absorptions. One sees that, for a fixed value of γ, k_2 is a multiple-valuated function of ν, which takes on two values for a given value of ν; one of these values is the negative of the other. This is the expected results because Eq. (10) is unchanged under the substitution $k\rightarrow -k$. Of course, the propagation direction of modes corresponding to positive values of k_2 is opposite to that of modes associated with negative values. For $\gamma=0$ and for weak and moderate absorptions, $k_2(\omega)$ exhibits both a maximum and a minimum at $\nu=\nu_C$, whereas it is a slowly varying function of ν in the limit of strong absorption. Hence, for the considered values of the absorption parameter γ, we may consider that $dk_2/d\omega=0$ at $\nu=\nu_C$. So, taking into account the latter condition as well as Eq. (11), it is not difficult to obtain the expression (for $\gamma\neq 0$)

$$\omega-\omega_C=\pm\frac{\pi\sqrt{f_1^2(\omega_C)-1}}{|df_2/d\omega|_{\omega=\omega_C}}|\tau| \tag{12}$$

for the mentioned asymptotic solution of Eq. (10), where the plus and minus signs correspond to the frequency ranges $\omega>\omega_C$ and $\omega<\omega_C$, respectively. Thus, around the center of the Brillouin zone, the frequency $\omega(k_1)$ is essentially a linear function of the Bloch wave vector k_1 and, therefore, the group velocity is practically independent of k_1.

As discussed before, the DOS plays an important role in the description and understanding of the effects of absorption on the zero-$\bar{n}$ gap. Due to this, we now present a detailed

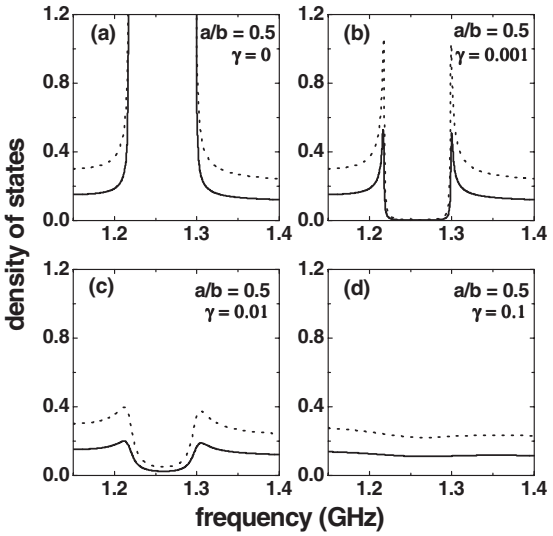

FIG. 4. Evolution of the density of states with the absorption parameter γ, for $a/b=0.5$, with $a=6$ mm (solid lines) and $a=12$ mm (dotted lines). Calculations are performed by using $\varepsilon_2(\omega)$ and $\mu_2(\omega)$ given by Eqs. (8) and (9).

study of such a quantity for the considered photonic crystals. In our approach, we may define the DOS as the number of wave vector k_1 per unit cell of the crystal for a given frequency ω,

$$\rho(\omega)=\frac{1}{N}\sum_{k_1}\delta[\omega-\omega(k_1)], \tag{13}$$

where N is the number of unit cells in the photonic crystal. Replacing the summation over k_1 by an integration in k_1 space and using Eq. (10), it is not difficult to obtain the following expression for $\rho(\omega)$ in terms of $f=f_1(\omega)+if_2(\omega)$:

$$\rho(\omega)=\left|\mathrm{Re}\left(-\frac{1}{2\pi}\frac{df/d\omega}{\sqrt{1-f^2}}\right)\right|. \tag{14}$$

In Figs. 4 and 5, the DOS obtained from Eq. (14) is plotted as a function of the frequency $\nu=\omega/2\pi$, for the same parameters a/b and γ as in Figs. 1 and 2, and in a frequency range where the corresponding zero-$\bar{n}$ gap is localized. In Fig. 4, we displayed the results for $a=6$ mm (solid lines) and $a=12$ mm (dotted lines), whereas in Fig. 5, $a=12$ mm (solid lines) and $a=18$ mm (dotted lines).

We first note that, ignoring quantitative difference, the properties of $\rho(\omega)$ and its evolution with the absorption parameter γ, for $a/b=0.5$ and 2, are very similar. As expected, in the absence of absorption, $\rho(\omega)$ is equal to zero inside the zero-$\bar{n}$ gap and is infinite at both edges of it [see Figs. 4(a) and 5(a)]. As is clearly seen in panels (b)–(d) of Figs. 4 and 5, these singularities are removed in the presence of absorption and $\rho(\omega)$ takes finite values inside the corresponding gaps. A similar phenomenon occurs in photonic crystals with conventional photonic band gaps (see Ref. 15). We also observe that, for weak and moderate absorptions, $\rho(\omega)$ and therefore the corresponding dispersion relation exhibit a pseudogap structure, which tends to disappear in the limit of strong absorption. In the latter case, $\rho(\omega)$ is practically inde-

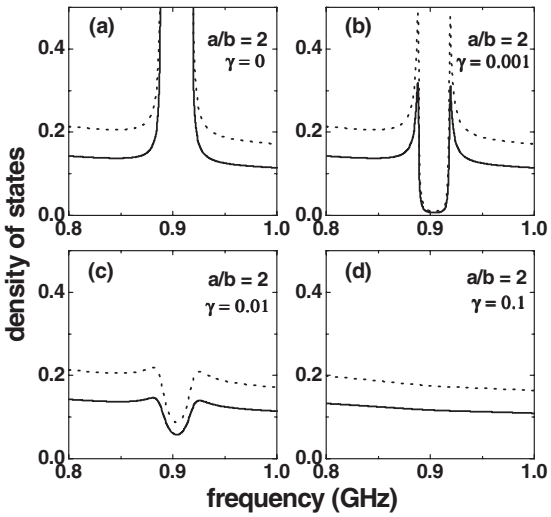

FIG. 5. The same as Fig. 4, except that $a/b=2$, with $a=12$ mm (solid lines) and $a=18$ mm (dotted lines).

pendent of ω in the considered frequency range [see Figs. 4(d) and 5(d)]. This is the expected result because, according to Eq. (12) and Figs. 1(d) and 2(d), the frequency $\omega(k_1)$ is essentially a linear function of the Bloch wave vector k_1 and, therefore, $d\omega/dk_1$ is practically independent of k_1. Further, as mentioned above, absorption creates photon states inside the band gap, giving rise to a redistribution of such states in the system. Results depicted in Figs. 4 and 5 reveal that such a redistribution of states involves mainly those states for which ω is near the band edges (for $\gamma=0$).

It is interesting to note that, using Eq. (14), we can rewrite the dispersion relation in Eq. (12) as $\omega-\omega_C=\pm|\tau|/2\rho(\omega_C)$; as a result, the corresponding group velocity $v_g(\tau)$ is proportional to the reciprocal of the density of states at $\omega=\omega_C$.

Let us briefly compare the above results with those obtained when another type of dispersion for both $\varepsilon_2(\omega)$ and $\mu_2(\omega)$ is used in Eq. (10) to model the LHM component of the photonic crystal. In order to perform the calculations we use the effective physical parameters,

$$\varepsilon_2(\nu) = 1 + \frac{5^2}{0.9^2 - \nu(\nu + i\Gamma_1)} + \frac{10^2}{11.5^2 - \nu(\nu + i\Gamma_2)}, \quad (15)$$

$$\mu_2(\nu) = 1 + \frac{3^2}{0.902^2 - \nu(\nu + i\Gamma_m)}, \quad (16)$$

which have been frequently used (see Refs. 5, 9, and 20) to studying the properties of the zero-$\bar{n}$ gap in the absence of absorption. In these equations, $\nu=\omega/2\pi$ is the frequency measured in gigahertz and Γ_1, Γ_2, and Γ_m are absorption constants, which, for simplicity, we take as $\Gamma_1=\Gamma_2=\Gamma_m=0.9\gamma$, where γ is a dimensionless absorption parameter. Figure 6 displays the associated photonic bands and DOS in a frequency range where the corresponding zero-$\bar{n}$ gap is localized. Comparing these results with those depicted in Figs. 1, 2, 4, and 5 we note that they show essentially the same physical behaviors.

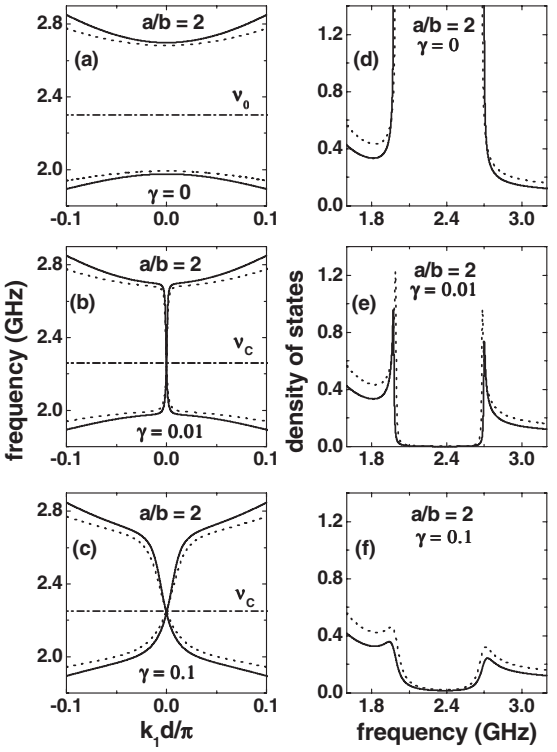

FIG. 6. Dispersion curves and density of states of a photonic crystal composed of alternating layers of air and a LHM with $\varepsilon_2(\nu)$ and $\mu_2(\nu)$ given by Eqs. (15) and (16), respectively, for $a/b=2$, with $a=12$ mm (solid lines) and $a=16$ mm (dotted lines), and for different values of the absorption parameter γ. ν_C satisfies Eq. (11) and ν_0 is the frequency at which the zero-$\bar{n}$ gap opens.

IV. CONCLUSIONS

In this paper we have investigated the effects of absorption on the properties of the zero-$\bar{n}$ gap in 1D photonic crystals consisting of alternating layers of air and a metal-based LHM. In this study, we calculated the photonic band structure and the corresponding DOS by using two different models for the physical parameters $\varepsilon_2(\omega)$ and $\mu_2(\omega)$ characterizing the LHM component. It is demonstrated that the band structure of the considered photonic crystals around the zero-$\bar{n}$ gap, which is insensitive to the geometrical scaling of the structure in the absence of absorption, is drastically modified by these effects. Specifically, it is shown that absorption creates photon states inside the gap and the resulting band structure exhibits a band gap of zero width. Another interesting consequence of these effects is that, for weak and moderate absorptions, the dispersion relation and the corresponding DOS exhibit a pseudogap structure which tends to disappear in the limit of strong absorption. We stress that our analysis can be readily extended in order to treat the effects of absorption on the remainder gaps of the considered system or on the band gaps of any other 1D photonic crystal for which its dispersion relation satisfies a transcendental equation similar to Eq. (10). According to the latter equation, the dispersion relation of any one of these 1D photonic crystals only exhibits band gaps of zero width in the presence of

absorption. Note that the frequencies at which the corresponding bands touch each other are the zeros of $f_2(\omega)$ [see Eq. (11)], i.e., Eq. (11) is the null-gap condition in the presence of absorption.

ACKNOWLEDGMENT

One of the authors (M.D.-L.) would like to thank R. L. Soto Morán for critically reading the paper.

[1] K. Sakoda, *Optical Properties of Photonic Crystals* (Springer, Berlin, 2001).
[2] J. D. Joannopoulos, P. R. Villeneuve, and S. Fan, Nature (London) **386**, 143 (1997).
[3] S. A. Ramakrishna, Rep. Prog. Phys. **68**, 449 (2005).
[4] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[5] J. Li, L. Zhou, C. T. Chan, and P. Sheng, Phys. Rev. Lett. **90**, 083901 (2003).
[6] L. Wu, S. He, and L. Shen, Phys. Rev. B **67**, 235103 (2003).
[7] D. Bria, B. Djafari-Rouhani, A. Akjouj, L. Dobrzynski, J. P. Vigneron, E. H. El Boudouti, and A. Nougaoui, Phys. Rev. E **69**, 066613 (2004).
[8] H. Jiang, H. Chen, H. Li, and Y. Zhang, Appl. Phys. Lett. **83**, 5386 (2003).
[9] H. Daninthe, S. Foteinopoulou, and C. M. Soukoulis, Photonics Nanostruct. Fundam. Appl. **4**, 123 (2006).
[10] N. C. Panoiu, R. M. Osgood, Jr., S. Zhang, and S. R. J. Brueck, J. Opt. Soc. Am. B **23**, 506 (2006).
[11] I. V. Shadrivov, A. A. Sukhorukov, and Y. S. Kivshar, Appl. Phys. Lett. **82**, 3820 (2003).
[12] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[13] R. A. Shelby, D. R. Smith, S. C. Nemat-Nasser, and S. Schultz, Appl. Phys. Lett. **78**, 489 (2001).
[14] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[15] A. A. Krokhin and P. Halevi, Phys. Rev. B **53**, 1205 (1996).
[16] V. Kuzmiak and A. A. Maradudin, Phys. Rev. B **55**, 7427 (1997).
[17] L. M. Li, Z. Q. Zhang, and X. Zhang, Phys. Rev. B **58**, 15589 (1998).
[18] V. Yannopapas, A. Modinos, and N. Stefanou, Phys. Rev. B **60**, 5359 (1999).
[19] M. M. Sigalas, C. M. Soukoulis, C. T. Chan, and K. M. Ho, Phys. Rev. B **49**, 11080 (1994).
[20] S. B. Cavalcanti, M. de Dios-Leyva, E. Reyes-Gómez, and L. E. Oliveira, Phys. Rev. E **75**, 026607 (2007).
[21] A. Tip, A. Moroz, and J. M. Combes, J. Phys. A **33**, 6223 (2000).
[22] N. Engheta and R. W. Ziolkowski, IEEE Trans. Microwave Theory Tech. **53**, 1535 (2005).